The Metaphysics of Mixed Quantities

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Abstract

Representationalism is a metaphysical theory of quantities which explains the fact that we use unit-relative numbers to represent quantities by appealing to intrinsic, non-numeric relations between individual quantities plus a representation theorem. While the theory is well-developed for quantities such as mass or length, that development has not been extended to "mixed quantities" such as kilogram meters per second or cubed meters per kilogram-seconds-squared. Since typical instances of nomic constants are given in mixed units, this lack of development has gotten tangled up in issues about the metaphysics of constants, including David Baker's "Pandora" argument against comparativism about quantities. This paper develops a representationalist theory of mixed quantities and explores how it can be used, in tandem with another thesis, to answer Baker's Pandora argument.

In 1959, *Luna 1* became the first man-made object to escape Earth's orbit. How was it able to do that? Partly because its velocity exceeded 11.18 km/s, the escape velocity a rocket needs to escape earth's gravity.

To a first approximation, this looks as though *Luna 1* succeeded by bearing a relation — the "velocity in km/s" relation — to a *number*. But while the number-using explanation isn't exactly *wrong*, it's natural to wonder whether the number could somehow be eliminated from the explanation. In other words, it's natural to wonder whether the number is needed for the *ultimate* explanation of the *Luna 1*'s success.

A "yes" answer is uncomfortable twice over. First, as Hartry Field (1980: 43, 2016: P-4, and 1984: 192–193) has emphasized, it's very odd to think that the rocket only knew what to do by consulting some abstract objects. The ultimate explanations of physical transactions ought to be **intrinsic**, not dragging in extraneous, non-physical props. (See also Jacobs (2022).) The *Luna 1*'s success should be, at least in principle, explicable by appeal just to physical objects, not physical objects plus some mathematicalia.

Second, quite aside from concerns about intrinsicality, a "yes" answer faces a dilemma. Did *Luna 1* succeed *just* by bearing the velocity-in-kilometers-per-second relation to 11.18? What about bearing the velocty-in-miles-per-minute relation to 9.5? Furlongs per year? Cubits per nanosecond?

There are ever-so-many possible systems of units. Each one gives rise to a relation between obejcts and numbers, and no such relation looks better-suited than any other to explain the *Luna 1*'s success. It would be unacceptably arbirtary to use just one of them in the ultimate explanation, and bizzarely profilgate to use them all.

Let a **quantity** be whatever we are talking about when we use expressions like "five pounds" or "11.8 km/s". **Representationalism** aims to resolve these issues about the metaphysics of quantities. Robert Stalnaker (1984: 9) summarizes the basic representationalist idea nicely:

> What is it about such physical properties as having a certain height or weight that makes it correct to represent them as relations between the thing to which the property is ascribed and a number? The reason we can understand such properties — physical quantities — in this way is that they belong to families of properties which have a structure in common with the real numbers. Because the family of properties which are *weights* of physical objects has this structure, we can (given a unit, fixed by a standard object) use a number to pick a particular one of the properties out of the family.

For responsible representationalists, Stalnaker's quote is the beginning, not the end, of the matter, for we need to say just what this "shared structure" *is*. And representationalists have taken up the charge, drawing heavily on work from measurement theory to characterize structures on quantities that make them well-represented by numbers, up to a choice of unit.[1]

Relatively less attention has been paid to the metaphysics of **mixed quantities**. We think of stereotypical quantities as the sorts of things that are assigned numbers relative to "feet" or "kilograms" or "seconds". But what about the kinds of things that are assigned numbers relative to "meters per second" or "meters cubed divided by kilograms-times-square-seconds"? What are we doing when we attach numbers to *these* units?

Laws that use fundamental nomic constants make the issue especially pressing. Newton, for instance, gives us as a gravitational law

$$F = G\frac{m_1 m_2}{r^2}.$$

In SI units, G is about $6.67 \times 10^{-11}\ m^3/kg \cdot s^2$. But what, from a *representationalist* viewpoint, does this amount to? The representationalist tells us what "5 kg" is supposed to mean; the next step is to tell us what "$6.67 \times 10^{-11}\ m^3/kg \cdot s^2$" means, too.

1 Representationalism

Units are, essentially, ratios. To call x seven feet long or pi kilograms massive is to say that it takes seven of the lengths we call "one foot" or pi of the masses we call "one kilogram" to make x's length or mass.

This isn't to say that *quantities* are ratios. A fifty-degree Farenheit day isn't, in any coherent sense, fifty times hotter than a one-degree-Farenheit day. But there is

[1] While Field (1980) is a clear early advocate of such a view, Mundy (1987) provides the representationalist's *locus classicus*; Wolff (2020) gives a recent and thorough development and defense.

a ratio in the neighborhood: the ratio of the "distances" between two temperatures. The difference between $2^\circ F$ and $4^\circ F$ really is half of the difference between $4^\circ F$ and $8^\circ F$. That's why we can measure temperatures with units: the degrees "count off" the distances (in that set of units) from some arbitrarily chosen zero point.

Where does this ratio structure come from? Tradition illustrates the idea with mass, which makes a nice example because we know two things about the masses.

First: They are **linearly ordered**. If a and b are masses, either one is greater than the other or they are equal. We use "$a \preceq b$" to say that a is no larger than b, and "$a \prec b$" to say that a is strictly smaller than b.

Second: Masses "combine," as it were. Two individual objects have a combined mass; if you put them on one side of a balancing scale, it would balance against some single object on the other. We call the combined mass a **concatenation**; if a and b are masses, we write this as $a \circ b$.[2]

So long as these masses satisfy certain highly plausible principles, we can prove a **representation theorem:**

Theorem. (Hölder)

There is a unique **ratio function**, f, which takes pairs of masses to numbers and satisfies, for any masses a, b, and u:

(i) $f(a,u) \leq f(b,u)$ iff $a \preceq b$;

(ii) $f(a \circ b, u) = f(a,u) + f(b,u)$; and

(iii) $f(u,u) = 1$.[3]

When we pick a unit, we pick a mass to fill in the u slot in this function; relative to that choice, f assigns each mass a unique real number. When we say "π kilograms", we're referring to the mass m where, if k is the mass we call "one kilogram", $f(m,k) = \pi$.

[2]This structure is the hallmark of **extensive quantities**. As Wolff (2020: §3.3) has emphasized, not all quantities of scientific interest are of this sort; as noted, temperature isn't. But Wolff goes on to argue that if non-extensive quantities can be given units they must have a "super-ratio structure" (2020: §6.2.3). In this case, *something* (such as "temperature difference") will concatenate, although this something might not be immediately detectable by any simple empirical operation.

[3]The theorem is originally due to Otto Hölder (1901); Michell and Ernst (1996) provide a partial translation. Dewar (2021) provides an elegant proof in an appendix, and I sketch it below in §5.1. A variant relying on weaker axioms is given by Krantz (1971).

The theorem is usually stated thusly: There is a function f from masses to numbers where $f(a) + f(b) = f(a \circ b)$, $f(a) \leq f(b)$ iff $a \preceq b$, and if g is any other function meeting these conditions, $g(x) = \alpha f(x)$ for some positive real α. It isn't difficult to see that my formulation entails this one, and that this latter one entails my formulation provided the masses are Dedekind-complete.

1.1 Choice Points

The view, thus specified, leaves many questions open about the nature of quantities. Different answers to these questions lead to alternative developments of representationalism.

The first question: What kinds of things are these quantities?

One possible answer: The masses are *properties* that things can have. Another possible answer: The masses are *the massive things themselves*. According to the first answer, if I'm twice as massive as you, that's because you instantiate a mass property m where I instantiate $m \circ m$. According to the second answer, if I'm twice as massive as you, that's because I am a concatenation of you with yourself.

There is a third answer as well: Masses might be *points in a space*. Structurally, this is quite like having masses as properties; we can swap the word "property" for "point" and "instantiates" for "occupies" to move between the views. (Cf. Arntzenius and Dorr, 2014: 228–230) This may have consequences for how we think about other metaphysical issues (Wolff, 2020: §7.3.2), but I won't make any hay over it here. Rather, I'll lump the property view and the mass-space view together under the term **substantivalism**, and the alternative under the term **relationalism**, words meant to evoke the Clarke-Leibniz debate. The idea is that, just as relationalists about space eschew spatial points in favor of spatial relations between concrete objects, relationalists about quantities eschew points of "mass-space" (or mass properties) in favor of ordering- and concatenation-like relations between concrete objects.

Relationalists about quantities face several problems, all related to the worry that there won't be enough massive things to go around (Field, 1984; Batitsky, 1998; Eddon, 2013*b*; Arntzenius and Dorr, 2014; Wolff, 2020). Since I've no reason to think that they'll be able to solve them, I'm going to assume substantivalism from here on in. That said, I suspect that, *if* relationalists can solve their other problems, they'll be able to use the materials developed here to solve problems of mixed units, too.

The second, and for our purposes, crucial question: *Why* are the masses ordered as they are, and why do certain ones count as the concatenations of others? Here are two possible answers.

One: The relationalist's higher-order relations are internal, holding thanks to the intrinsic natures of the relata. *This* mass is no larger than *that* mass because of *what the mass properties are like*, in and of themselves.

Two: the higher-order relations are in a sense something "extra", above and beyond whatever intrinsic natures the mass properties might have (if any).

These two answers map nicely onto two alternative views about what's fundamental. According to the first, the *mass properties themselves* are fundamental, and the comparative relations between them derivative. According to the second, it is the *comparative relations*, rather than the mass properties, that are fundamental.

It is tempting to call the first view 'absolutism' and the second one 'comparativism,' since the first thinks that absolute quantities are fundamental and the second thinks that comparative relations are instead fundamental (Wolff, 2020). But usage is tricky here, as those terms have already been coined by Shamik Dasgupta (2013), who seems to intend by 'comparativism' what I have called *relationalism*. As a result, usage isn't standardized.

Consider a substantivalist view on which mass properties are thought of as analogous to points of a space, with no internal structure of their own; everything interesting about mass properties, on this view, depends on and only on the fundamental comparative relations that hold between them. As Wolff (2020) uses the terms, for instance, this view counts as "comparativist". But other authors (e.g., Shumener (2022)), following Dasgupta more closely, would count it as "absolutist".

I find Wolff's usage helpful, so I will follow it here; others, of course, may use the terms as they see fit. I only ask that they don't thereby misinterpret me. More specifically, here's how I'll taxonomize: **strong comparativism** is the conjunction of relationalism with the claim that the comparative relations are fundamental (and individual quantities are not). Substantivalists are then divided into the **absolutist** and **comparativist** varieties, comparativists holding that only the comparative relations are fundamental, and absolutists thinking the reverse.[4]

Note that comparativists and absolutists are equally committed to either the structuring relations or the quantities themselves being fundamental, *but not both*. A third option is also available: **mixed absolutism**, which takes both the quantities and the relations as fundamental (Sider, 2020: 136). Maya Eddon (2013*a*: §5) argues that trying to build relations out of quantities' intrinsic natures will lead to problematic underdetermination. If that's right, would-be absolutists may take refuge in mixed absolutism instead.

2 A Problem for Comparativism

2.1 *Pandora*

David Baker (2020) raised the following problem for comparativists.[5] Suppose that the *Luna 1* only barely made it out of orbit, its velocity just reaching 11.18 km/s. Let t be a time shortly after its liftoff, and consider a possible world with the same laws of nature where, at t, everything is precisely as it actually is except all of the masses are doubled. Call the first world "Earth", and the second, "Pandora".

On the one hand, Pandora's *Luna 1* ought to crash. After all, in order to make it

[4]There's a spot on the grid for combining relationalism with the claim that the intrinsic natures of the relata are what is fundamental, but I have a hard time seeing how to keep that from collapsing into some sort of substantivalism–what are these "intrinsic natures" if not properties– so I won't bother naming it here.

[5]In fact, Baker only had strong comparativism in his sights, but the argument can be extended, as we will see.

into orbit, the rocket has to reach a velocity given by the escape law

$$v_p = \sqrt{\frac{2GM_p}{r_p}},$$

where G is the gravitational constant and M_p and r_p are the respective mass and radius of Pandora. Since the masses (but not the planet's radius) are doubled in Pandora, this will be $\sqrt{2}$ times as much as v_a, the velocity the actual *Luna 1* needs to make it into Earth's orbit. But since the actual *Luna 1* only barely made it into orbit (we are supposing), and Pandora's hits the same velocity, the latter will crash.

The problem is that, if comparativism is true, then Pandora and the actual world would seem to be fundamental duplicates at t. Doubling all the masses shouldn't change the distribution of $\preceq$ and $\circ$; if your mass is actually three-quarters of mine, then in Pandora your (doubled) mass is still three-quarters of my (also doubled) mass. Since (according to comparativism) only these comparative relations are fundamental, the worlds don't differ in their distrubution of fundamental properties and relations. Yet if the worlds are fundamental duplicates at t, have the same laws, and evolve in different ways, we have a rather distressing failure of determinism in what was meant to be a paridigmatically deterministic theory.

2.2 *The Argument Refined*

It's easy to suspect that the argument just given smuggled in some untoward assumptions. To ward off such suspicions, let's present it more carefully.

Before we do, let's address one initial objection. The comparativist might insist that, by their lights, talk of "doubling all the masses" makes no sense, in which case our description of Pandora does not describe a possible world and so we have no failure of determinism.

As Niels Martens (2021 and 2022: 333-334) stresses, though, denying the existence of a Pandora-like world is scientifically revisionary. According to Newtonian gravitation as she is played, there are two possible worlds, w_1 and w_2, where at t all the physical quantities are assigned the same unit-relative numbers, except that the numbers assigned to masses in w_2 are doubled. Newtonian gravitation treats both descriptions as picking out nomically possible world-states, embedded in nomically possible worlds which differ as to what happens to the rocket. Unless comparativists want to go against the science, they ought to countenance at least this much.

We'll use this observation as our starting point: There are worlds w_1 and w_2 — Earth and Pandora — that admit number-using descriptions of just this sort and that obey the same laws of nature.

Before going on, we should also make precise the relevant notion of "fundamental duplicate." First, let a **fundamorphism** between two worlds be a one-to-one correspondence between the denizens of each world that preserves fundamental properties and relations. Worlds that admit a fundamorphism are **fundamental duplicates**.

In general, fundamorphisms will preserve more than just fundamental relations. Anything definable from the fundamental, for instance, will also be preserved. Lewis bases his (1983) definition of "intrinsic" properties and relations off this observation. Since this use of "intrinsic" is not obviously the same as Field's notion invoked above, to avoid confusion I'll call properties and relations preserved by fundamorphisms **F-intrinsic**.[6]

I also assume that determinism is to be understood in terms of fundamental duplication. As a first pass, some laws are deterministic if and only if, if w and v are worlds with those laws and if w and v are fundamental duplicates at some time, they are fundamental duplicates at *all* times.[7]

With that in mind, let's go back to our worlds w_1 and w_2. Since we're assuming comparativism, the fundamental facts that license these worlds' numerical descriptions must hold in virtue of fundamental comparative relations. Now, it's trivial to map every quantity in Earth to the quantity in Pandora that is assigned the same numerical value in a given system of units. This gets us a function g with these properties:

- For any object x and non-mass quantity q, x instantiates q at t iff $g(x)$ instantiates $g(q)$ at t.
- For any object x and mass quantity q, x instantiates q at t iff $g(x)$ instantiates $g(q) \circ g(q)$ at t.

Next, we take a function h which maps each mass property to its self-concatenation and leaves everything else alone. The axioms needed for the representation theorem can be used to prove the following:

- $a \preceq b$ iff $a \circ a \preceq b \circ b$.
- $a \circ b = c$ iff $(a \circ a) \circ (b \circ b) = c \circ c$.

Since $h(a) = a \circ a$, this gives us:

- $a \preceq b$ iff $h(a) \circ h(b)$.
- $a \circ b = c$ iff $h(a \circ b) = h(c)$.

[6] Lewis uses "perfeclty natural" rather than "fundamental", and as he doubts that parthood is perfectly natural, he adds a clause that in a fundamorphism parthood is also preserved. I follow Eddon (2017) in taking parthood as fundamental, and need no such clause.

[7] Why "at a first pass"? Because we may think that, e.g., instantaneous velocities are not F-intrinsic to times, but instead depend on arbitrarily small temporal regions around times, in which case it is not fundamental duplication at a time that matters for determinism, but fundamental duplication at some arbitrarily small but extended temporal interval. As Baker (2020) makes clear, this undermines the original Pandora case, but not other, similar cases. Since the Pandora case is easy to think about, I'll stick with it here.

So, finally, we define $f(x) = h(g(x))$. We've seen that an object x will instantiate a quantity q at t on Earth iff $f(x)$ instantiates $f(q)$ at t in Pandora, and f will preserve the higher-order structuring relations that hold betwen the quantities. So f is a fundamorphism. Thus Earth and Pandora are duplicates at t, despite obeying the same laws and having different futures. So we have a failure of determinism.

Of course, given substantivalism, f will mess with which objects instantiate which mass properties. But that doesn't matter, because the central thesis of comparativism is that *those* facts aren't fundamental. My instantiation of some particular mass property isn't a comparative fact about mass, and so needn't be preserved to make worlds fundamental duplicates.

Notice a few things about this argument. First, we made no assumptions about whether the masses were "really" doubled, or whether it even made sense to ask whether quantities were the *same* across worlds. What mattered was only that the two worlds, as described by Newtonian gravitation, were possible, and that the function between them was a fundamorphism.

Second, we made no assumptions about whether or not fundamental duplication of worlds implied their identity. One pressing issue in the metaphysics of physics involves a theory's *symmetries*. The issues can get technical, but the underlying concern is about whether the theory says there are distinct states that are nonetheless "physically equivalent". Plausibly, physical equivalence implies fundamental duplication. By taking no stand on whether distinct worlds can be fundamental duplicates, we took no stand on these thorny issues .

Third, the absolutist — mixed or otherwise — is not on the hook for this argument. According to her, the *individual mass properties* are also fundamental, and so must be preserved by a fundamorphism. Thus, to be a fundamorphism, f must map M to M. And it doesn't.

Sider (2020: 159) has argued that, despite this, absolutists are not obviously better-off. While they do not think that Earth and Pandora are fundamental duplicates, given the overall structural similarities between the two, we're left wondering how the laws "know" what the rocket should do. The worlds only differ quidditistically, and it's unclear how the laws distinguish worlds with mere quidditistic differences of this sort.

Arguments of this sort will seem compelling to the extent we think that the laws can only respond to broadly structural features, which I take to be one of the key issues in this debate. Rather than take a stand on the issue, I'll set absolutism aside for now; we'll return to it in §6.

2.3 Towards a Solution

One repsonse to the argument takes issue with the details of the case. Another accepts that there is a failure of determinism (as we defined it), but argues that this is not a problem, as a broader kind of "determinism" (in which the state of the world at one moment depends on the future) is all we should have hoped for in the first place (Dasgupta, 2020).[8]

These strike me as desperate moves. We can do better. In response to his own argument, Baker suggests (in essence) that maybe our constructed f wasn't a fundamorphism after all, *even by comparativist lights*. There was another fundamental relation that we missed out on, and once we see what it is, we'll see that f doesn't preserve it.

What would this relation be? Start by considering this response to our first, sketchy version of the argument:

> You said that, in Pandora, all the masses are doubled. But in that case, Pandora will have a *different* value for the gravitational constant, G. Because, after all, the units of G are $m^3/kg \cdot s^2$. If you double the masses, you have to do something about the "kg" in G, too![9]

Martens (2022: 334–335) argues forcefully and in detail against this sort of response. To see the force without the detail, note that, however intuitively compelling this speech might seem, it tells us nothing about where the argument went wrong. It doesn't challenge the idea that f is a fundamorphism or that Earth and Pandora are both physically possible.

Still, the fact that "kg" shows up in the gravitational constant, which is the law leading to Pandora-shaped problems, is suggestive. Somehow or other, the gravitational constant connects masses, distances, and durations, and when we double the masses in Pandora, we disrupt this connection. Caspar Jacobs (2023: 803) describes the constant as an "exchange rate" between these quantities. We can think of it as a fundamental relation, *Grav*, relating them. When we do, we see that, while f might preserve the other fundamental properties and relations, it does *not* preserve *Grav*.

While this response has been suggested by Baker and Jacobs (2023),[10] questions remain. What kind of thing is this "relation"? How complicated is it? How new or novel is the structure needed to make it work? What are we committing ourselves to, metaphysically, by going down this route? The proposal needs more development.

One important fact noted by Baker and Jacobs is that the gravitational constant is a "mixed quantity" — a quantity that is essentially tied up with other quantities. This is what lets it connect mass with other quantities in a way that gets disrupted in Pandora. Thus we arrive back at our main topic: Developing a representationalist account of mixed quantities.

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[8] The dialectic is commplicated by the fact that Dasgupta defends *strong* comparativism.

[9] I have heard an argument of this sort attributed to Roberts (2016).

[10] Jacobs calls G a "complex piece of real cross-value space structure" (805), but also distinguishes his view from Baker's by saying that, while Baker takes interquantity relations to be fundamental, he takes them to be "induced" by a constant of nature" (805 n. 12). I confess to not understanding what a "real piece of cross-value space structure" that "induces" a relation but is not itself a fundamental relation is supposed to be.

In the next section, I start developing one by working through a handful of toy cases. Before that, however, it's worth saying a little more about just what we want this account to do.

First, I assume we have a fixed stock of **basic** quantities — for current purposes, mass, length, and duration will suffice — and the "mixed" quantities are somehow built out of these.[11]

There are of course chocies to be made. Consider force, for instance. It has (we say) units of one Newton, or one kilogram meter per second squared. When we say "Newton", it sounds as though force is a basic quantity. When we say "$kg \cdot m/s^2$", it sounds like a mixed quantity. Which one is it?

It depends on whether we think force is basic or not. If it isn't, then fundamental comparative relations hold between (say) masses, and between durations, etc., but not between any thing we'd call a "force property". Instead, forces are built up out of these. If force *is* basic, then there are force properties with fundamental comparative relations holding between them. Once we see what the options are, we needn't take a stand on the issue.

According to the second view, force isn't a mixed quantitiy at all, but another entry in our list of basic quantities. This might tempt us to think we need no metaphysics for mixed quantities. But that would be an error. Consider Newton's second law:

$$F = ma.$$

If we think of force as a mixed quantitiy, built out of mass and acceleration, then this law can be understood as something in the neighborhood of a defintiion, telling us what kind of mixed quantity force is. But if force is its own separate kind of quantity, this law is really short for another one,

$$F = Ama,$$

where A is an "acceleration constant" telling us how masses and accelerations hook up to forces. When we represent our quantities in SI units, this constant has the value of $1N \cdot s^2/kg \cdot m$, so we don't write it down. But in other sets of units this constant may have a different value. And *this constant* will represent a mixed quantitiy — one built up out of mass, duration, distance, and force. Taking forces as basic doesn't get rid of the need for mixed quantities, but changes which mixed quantities we need to worry about.

My next assumption is a standard representationalist story for the basic quantities. I'll assume they are structured by fundamental ordering ($\preceq$) and concatenation

[11]Don't confuse "basic" with "fundamental". Basic quantities might in some sense be *relatively* fundamental — more fundamental than the mixed quantities built out of them — but that doesn't mean they're *fundamental*, full stop. Compare with LEGO bricks; the bricks are in a clear sense the "basic" ingredients of a LEGO castle, but that doesn't mean the LEGO corporation's factories are creating fundamental entities.

($\circ$) relations, and that (via suitable representation theorems) each quantity family has a class of representation functions that assigns numbers to the quantities relative to units — "unit functions", for short. For ease of presentation I'll use a unit's abbreviation for its corresponding unit function. For instance, "kg" denotes the kilogram unit function, and "$kg(a)$" the number that this function assigns to the property a.

With this in mind, we want our account to do three things. First, there should be a natural way to assign, relative to any choice of units for the basic quantities, a unique number that representes the mixed quantities in those units. Second, this number should re-scale appropriately as we change units; the recipe that assigns about $6 \times 10^{-11} m^3/kg \cdot s^2$ to whatever the gravitaitonal constant is had better also assign about $6 \times 10^{-14} m^3/g \cdot s^2$ to it. (Jacobs, 2023: 804). And third, the account should help explain how the actual world and Pandora could fail to be fundamental duplicates.

3 Some Toy Problems

Let's forget about Pandora for the moment and ask ourselves what the numbers in a physical constant represent. To make the question easier to think about, we'll start with some toy cases.

3.1 *X*-Fields and *Z*-Fields

Imagine that scientists learn how to create a special kind of field — an X-field — with the following features: An X-field can only ever have two iron spheres within it, but if the masses of the two iron spheres ever add up to one kilogram, the spheres vanish. This suggests a law of nature, one which has a constant, V^+:

The Additive Vanishing Law: When masses m_1 and m_2 are in an X-field, they vanish if and only if $m_1 + m_2 = V^+$.

The value of V^+ is one kilogram. Our task is to figure out what V^+ represents.

Since V^+ is assigned "one kilogram", it's tempting to identify the constant with this mass property. Before we do that, though, let's consider a modified case. A Z-fields is just like an X-field except that the spheres vanish when the *product* of their masses is one square kilogram. So they are governed by

The Multiplicative Vanishing Law: When masses m_1 and m_2 are in a Z-field, they vanish if and only if $m_1 \cdot m_2 = V^\times$,

where $V^\times$ is one square kilogram.

Even if we could identify V^+ with the one-kilogram-mass property, we can't identify $V^\times$ with that property, because it won't scale right when we change units. To see this, imagine two scientists studying Z-fields. One of them, working in kilograms, records two spheres vanishing with respective masses of .5 kg and 2 kg. The other, working in grams, records the same spheres as vanishing at respective masses of 500 g and 2000 g. So when they each use their findings to calculate the value of $V^{\times}$, the first get one square kilogram and the second, one million square grams. The property assigned "one" by the kilogram system of units, however, is assigned one *thousand*, not one million, by the gram system. So this mass property isn't a suitable candidate for $V^{\times}$. And, in fact, *no* mass property can be such a candidate, for there is no mass property a where $kg(a) = 1$ and $g(a) = 1$ million.

3.2 *Vanishing Pairs*

Although a particular mass property won't remain constant under change of units, there is something that will: the pairs of masses that vanish in a Z-field. More precisely, let

$$\textsc{Van} := \{(x, y) : \text{ if } x \text{ and } y \text{ are instantiated together in a } Z\text{-field their bearers will vanish}\}.$$

Van satisfies our first two desiderada. For each unit function u, there is a number n where, if $(a, b) \in \textsc{Van}$, $u(a)u(b) = n$. Furthermore, that number scales appropriately when we change units.

From the comparativist's perspective, though, Van isn't a great candidate to be the constant itself. The constant, as something integral to the laws of nature, ought to be fundamental (Lewis, 1983). According to the comparativist, the masses aren't fundamental, so a collection of mass pairs ought not be fundamental, either.

Relatedly, Van won't satisfy our third desiderata. We can run a Pandora-style argument involving the vanishing law, where on Earth two half-kilogram spheres don't vanish but on Sphere-Pandora their mass-doubled counterparts do. Since the mass pairs aren't fundamental, we ought to be able to construct a fundamorphism between the worlds, leading to another failure of determinism.

Rather than focusing on the mass pairs, we can identify the constant with a fundamental, higher-order relation, ZV, of which Van is the extension. Then, to say say that a number n represents ZV in a system of units u is to say that, for any masses a and b, $ZV(a, b)$ iff $u(a)u(b) = n$.

If we do this, we resolve our Pandora-style worries. Consider the condition:

> For some masses a and b: x instantiates a and y instantiates b and it isn't the case that $ZV(a, b)$.

This is an F-intrinsic condition; "a" and "b" aren't names of particular masses, but variables uses to quantify over mass properties. But while spheres x and y do not satisfy this condition, their counterparts in Sphere-Pandora do. Since the condition is F-intrinsic, the worlds are not fundamental duplicates.

3.3 *Intrinsic Laws*

If, as Field (1984: 188–189) and others (Arntzenius and Dorr, 2014: 215) have argued, we ought to search for fundamental intrinsic explanations, then we ought also search for intrinsic formulations of the fundamental laws — formulations that don't drag in anything extraneous (such as "unit functions"). The current proposal lets us do this. When it comes to Z-fields, we have:

Multiplicative Vanishing: If x_1 and x_2 are in a Z-field and instantiate masses m_1 and m_2, then they vanish iff $ZV(m_1, m_2)$,

which only uses fundamental notions.

Something similar holds for X-fields. To avoid Pandora-style concerns, we ought to identify V^+ with a fundamental higher-order property XV, and formulate the law as:

Additive Vanishing: If x_1 and x_2 are in an X-field and instantiate masses m_1 and m_2, then they vanish iff $XV(m_1 \circ m_2)$.

3.4 *Square Masses*

These vanishing laws explain the behavior of spheres in X- and Z-fields. But they aren't yet enough to tell us that XV or ZV are *constants*. We know that they are because we set the case up that way. But we haven't yet said what *makes* this so. What is the *world* doing to guarantee that ZV and XV can be represented by numbers, relative to a choice of units, or that once they are so-represented, they will scale properly?

The laws should predict that the constants are constant; the fact that we can use mixed-unit-relative numbers to represent them isn't a coincidence, after all.

As Jacobs (2023: 804) points out, a given constant's scaling pattern is itself an empirical matter. What do we learn when we discover that a constant has a particular scaling pattern? We learn what "type" of mixed quantity the law involves. The type of this mixed constant can then be given by a *second* law — a law we discover when we discover a constant's scaling pattern. For example, when we discover that spheres in a Z-field vanish at a product of their masses, we discover a law stating that the constant ZV is a "square mass".

We know how to formulate such a law for the additive constant XV: It says merely that XV is had by exactly one mass property. Since every mass property is represented by a number in each system of units, XV inherits this representation automatically.

Things are more complicated when it comes to ZV. There are lots of relations between masses, and many of them won't be, in any reasonable sense, representable by a number. The relation that holds between x and y when x is a mass instantiated by a U.S. President and y is a mass instantiated by x's Vice President, for instance, won't be so representable. It's not well-enough behaved.

Our law needs to guarantee ZV's good behavior. It must ensure two things: (i) that ZV is represented by some number for each choice of unit, and (ii) that when we switch units, the number scales appropriately.

Let's start with condition (i). This amounts, in the case of square masses, to every unit function u having a number n where

$$ZV(c, d) \text{ iff } n = u(c)u(d).$$

Notice that, if this *did* hold, we could pick some a and b where $ZV(a, b)$ and rewrite this condition as

$$ZV(c, d) \text{ iff } u(a)u(b) = u(c)u(d).$$

This is the version of the condition we'll work with.

Call a relation a **product** of a and b when it meets this condition, and write it as $P[a, b]$. We want a way of saying that a given relation is a product of two masses that doesn't appeal to this condition, but rather *entails* it.

Note first that the fundamental setup of representationalism — the tools needed to prove the representation theorem — gives us a four-place "ratio equivalence" relation,

$$a{:}b \equiv c{:}d,$$

which says, in effect, that the ratio beween a and b is the same as that between c and d. (More on this in §5.) Since units in fact express ratios, this has the nice consequence that, for any unit functions u and u',

$$\frac{u(a)}{u(c)} = \frac{u'(d)}{u'(b)} \text{ iff } a{:}c \equiv d{:}b.$$

So we can set u to u' and cross-multiply to conclude that

$$u(a)u(b) = u(c)u(d) \text{ iff } \frac{u(a)}{u(c)} = \frac{u(d)}{u(b)}.$$

Putting these two together, we can define a product of a and b as the relation $P[a, b]$ where

$$P[a, b](c, d) \text{ iff } a{:}c \equiv d{:}b.$$

We can then represent $P[a, b]$, relative to a set of units, by the number $u(a)u(b)$, and there will be just one such number for any product.

Even better, this already satisfies our condition (ii). Suppose we have two unit functions, u and v; then v will be the scaling of u by some constant α. In the system u, $P[a, b]$ is represented by $u(a)u(b)$. In the system v, it is represented by

$$v(a)v(b) = \alpha u(a)\alpha u(b) = \alpha^2 u(a)u(b).$$

But this is precisely what we want; a move from one mass unit to another that scales the first by α should also scale the *square* mass units by α^2.

Now that we know how ZV needs to work to satisfy conditions (i) and (ii), we can put it all into another law of nature:

$V^\times$ **is a Square Mass:** There are masses a and b such that $ZV(m_1, m_2)$ iff $P[a,b](m_1, m_2)$.[12]

4 Mixed Quantities

A "square mass" is one kind of mixed quantitiy. It is perhaps the simplest kind, because it is simply one quantity "multiplied" by itself. But we encounter much more complicated mixed quantities and need to deal with all of them.

In general, mixed quantities all involve "multiplying" and "dividing" basic quantities. (When we add or subtract basic quantities, we end up with the same sort we started with; only multiplication and division complicate things.) If we want to extend the example of mass products, we'll have to generalize in three directions. First, we'll have to allow for products of more than just two basic quantities. Second, we'll have to deal with something like division. And third, we'll need to allow multiplication and division of quantities from different families, so that (for instance) we can multiply a distance by a mass.

4.1 *Generalized Products*

Let's start by extending our notion of "mass product" to include products of any number of masses. Suppose we want a relation to serve as the product of masses a, b, and c. If we follow the strategy of §3.4, we'll start by noting that, for any system of units u and masses x, y, and z, saying that x, y, and z have the same mass product as a, b, and c would come to:

$$u(a)u(b)u(c) = u(x)u(y)u(z).$$

The we cross-multiply, getting

$$\frac{u(a)u(b)}{u(x)u(y)} = \frac{u(z)}{u(c)}.$$

To "translate" this into unit-free talk, we have to say

$$P[a,b]{:}P[x,y] \equiv z{:}c.$$

[12]Cian Dorr (2010) objects to laws formulated with existential quantifiers out front like this, for reasons that may interact with potential motivations for comparativism and/or intrinsic formulations of laws. So its worth noting that we could have done just as well with a law saying, "for any a and b, if $ZV(a,b)$, then for any c and d, $ZV(c,d)$ iff $P[a,b](c,d)$."

The problem is that this doesn't yet *mean* anything. The basic representationalist apparatus lets us make ratio comparisons between individual quantities. It doesn't let us make direct ratio comparisons between quantities *and products*.

There is a way to make indirect comparisons, though. Suppose we have mass products $P[a,b]$ and $P[x,y]$. For any mass v, there will be masses A and X where $P[a,b](A,v)$ and $P[x,y](X,v)$.[13] If we put this in a system of units u, we'd have

$$u(a)u(b) = u(A)u(v) \qquad\qquad u(x)u(y) = u(X)u(v)$$

and so we could rewrite the original equivalence as

$$u(A)u(v)u(c) = u(X)u(v)u(z).$$

Now when we cross-multiply, the $u(v)$s cancel, so we get

$$\frac{u(A)u(v)}{u(X)u(v)} = \frac{u(A)\cancel{u(v)}}{u(X)\cancel{u(v)}} = \frac{u(A)}{u(X)} = \frac{u(z)}{u(c)}.$$

This observation gives us a way to define ratios between products. The idea is that we replace the ratio of products with ratios of quantities which are proxies of this sort relative to a "normalizing" mass such as v.

We'll make things easier by defining a general "product-ratio" relation that we can reuse as needed. To avoid notational clutter, let's write lists of variables in boldface; $\boldsymbol{a}$, for instance, is shorthand for $a_1, \ldots, a_n$, where the length of the list is determined by context. Using this notation, we have:

$P[\boldsymbol{a}]{:}P[\boldsymbol{b}] \equiv P[\boldsymbol{c}]{:}P[\boldsymbol{d}]$ iff, for any $\boldsymbol{y}$ and $\boldsymbol{z}$, there are A, B, C, and D, where

(i) $P[\boldsymbol{a}](A, \boldsymbol{y})$,
(ii) $P[\boldsymbol{b}](B, \boldsymbol{y})$,
(iii) $P[\boldsymbol{c}](C, \boldsymbol{z})$,
(iv) $P[\boldsymbol{d}](D, \boldsymbol{z})$, and
(v) $A{:}B \equiv C{:}D$.

Next, we adopt the convention where, for a single quantity a, $P[a]$ is the property of being identical to a. (In this case, the schema $P[a](A, \boldsymbol{y})$ holds exactly when $A = a$.) Then we can use this as the base for an inductive definition of products:

[13] This is guaranteed by having as many masses as there are (positive) real numbers, something a relationalist cannot assume. This is thus one place where relationalists wishing to use the present apparatus will need to tread carefully.

$P[\boldsymbol{a}, b](\boldsymbol{c}, d)$ iff $P[\boldsymbol{a}]:P[\boldsymbol{c}] \equiv P[d]:P[b]$.[14]

4.2 Products from Different Families

Before thinking about division, let's consider products of quantities from *different* families. Energy, for instance, is a product of distance and force. If distance and force are both basic units, they will need products. If they are not, we will need other products to build up to them, but we will still need cross-family products. And examples multiply. The gravitational constant is in cubed distances per mass times seconds squared. We'll need to be able to make sense of masses times seconds squared in order to make sense of this more complex mixed quantitiy.

The surprising thing is that there is almost nothing extra we have to do. It turns out that our ability to define $a{:}b \equiv c{:}d$ doesn't depend on all four properties being of the same type. The properties a and b must be in the same family, as must the properties c and d. But the two pairs can come from different families.

It's easy to get suspicious of this. In considering a similar proposal, Baker remarks, "it sounds like nonsense to say that a planet is 'twice as wide as it is massive'" (Baker: 11).[15] But if m_1 and m_2 are masses and l_1 and l_2 are lengths, then $m_1{:}m_2 \equiv l_1{:}l_2$ says no such thing. One way for $m_1{:}m_2 \equiv l_1{:}l_2$ to hold is for m_2 to be twice m_1 and l_2 to be twice l_1. This doesn't in any sense compare the individual lengths to the individual masses. Rather, it compares the *structural relation between* the two lengths to the *structural relation between* the two masses. Since both lengths and masses have structural relations making it sensible to talk about them in terms of ratios, we shouldn't be shocked if these structural relations were themselves comparable.

Some will remain unconvinced, so I'll return to this issue in §5. If we spot it for now, we can see that our above definitions don't depend on the properties being all in the same family. They will only depend on the properties *on the same side of the* "$\equiv$" being in the same family. As a result, we can have $P[a, b]$ where a is a mass and b is a length, since this will hold of c and d only if c is a mass and d a length where $a{:}c \equiv d{:}b$. As this is only defined when a and c, on the one hand, and b and d, on the other, are co-familial, these restrictions fall out naturally from our definition.

[14] This way of proceeding might seem circular, since we defined our generalized equivalence in terms of products *before* our official definition of products themselves. To see why there's no real circularity, imagine starting with the base definition of $P[a]$, defining product equivalence for these "one-products", using that to define products of two masses, then defining equivalence for "two-products", then bouncing back and forth between the two definitions as we go up our definitional hierarchy.

[15] To be clear, Baker does not endorse this as an objection even against the view he is considering, which is not this one.

4.3 *Quotients*

Products are relations between mixed quantities, and can serve as number-free correlates of property-multiplication. But we also need number-free correlates of property-division. If Y-spheres vanish when the ratio of their masses to their diameters hit a certain point — one kilogram per meter, for instance — we'll need to know what this ratio represents.

If products are correlates of multiplication, then **quotients** will be correlates of division. If we have a mass m and a length l, for instance, the quotient between m and l will be the relation $Q[m/l]$ where

> $Q[m/l](a,b)$ iff $m{:}a \equiv l{:}b$.

Once we have quotients, we'll want to multiply and divide them by other properties, products, and quotients. Since division of a fraction is the same as multiplication by an inverse, we can do all of this just by having quotients of products. We can think of the product of quotients $Q[a/b]$ and $Q[c/d]$ as a quotient of products $P[a,c]$ and $P[b,d]$, and we can think of the quotient of $Q[a/b]$ with $Q[c/d]$ as the product of quotients $Q[a/b]$ and $Q[d/c]$.

So we need only define quotients of products. Recall that we can always replace any single quantitiy a by its "one-product", $P[a]$. As a result, the above definition of quotients of two individual quantities will emerge as a special case of our more general definition.

The quotient of products $P[\boldsymbol{a}]$ and $P[\boldsymbol{b}]$ is written $Q[\boldsymbol{a}/\boldsymbol{b}]$, and is defined:

> $Q[\boldsymbol{a}/\boldsymbol{b}](\boldsymbol{c},\boldsymbol{d})$ iff $P[\boldsymbol{a}]{:}P[\boldsymbol{c}] \equiv P[\boldsymbol{b}]{:}P[\boldsymbol{d}]$.

This uses the general notion of product ratio equivalence defined above.

Some nomic constants are "dimensionless". These constants don't re-scale with a change of units, and it is tempting to view them as some sort of super-powerful, mysterious thing, a cut above the usual "dimensioned" quantities.

Perhaps some are. But they needn't all be. On the present picture, a quotient between (say) two masses will be "dimensionless", because it will be represented by a ratio, and ratios are constant across unit changes. The units "cancel", as it were. But that needn't make it fundamentally different sort of thing. One's waist-to-height ratio is dimensionless, but it's still just a ratio of lengths, after all.

As a result, the mere fact that some constant is dimensionless doesn't mean it isn't amenable to the present treatment. For instance (*pace* Jacobs 2023: 797), the dimensionless fine structure constant may correspond to a quotient of energies. Dimensionless mixed quantities are simply quotients with the same sort of product on the top as on the bottom.

4.4 A Theory of Mixed Quantities

Here, finally, is a theory of mixed quantities. Given some basic quantities, a **mixed quantity** is any product or quotient of basic quantities, as defined above. (The degenerate products of the form $P[a]$ are **improper** mixed quantities; the rest of the products are **proper**.) A **total system of units** consists of one unit function for each type of basic quantity; we write it as a function of its own, u. Finally, for any total system of units u, we extend it to mixed quantities as follows:

- $u(P[a_1, \ldots, a_n]) = u(a_1) \cdot \ldots \cdot u(a_n)$, and
    - $u(Q[\boldsymbol{a}/\boldsymbol{b}]) = \frac{u(P[\boldsymbol{a}])}{u(P[\boldsymbol{b}])}$.

We should note a few things about these mixed quantities. First, the mixed quantities themselves come in families. Let the **signature** of a product $P[\boldsymbol{a}]$ be a specification of just which family each a_i comes from, and let the signature of a quotient be the ordered pair of the signatures of its respective products. A **family** of mixed quantities is a maximal collection of same-signatured quantities.

Each family of mixed quantities has a natural intrinsic structure that mirrors the concatenation-and-ordering structure of the basic quantities.[16] For instance, given products of the same signature, we can define

$P[a, \boldsymbol{b}] \preceq P[c, \boldsymbol{d}]$ iff for every $\boldsymbol{v}$, if $P[a, \boldsymbol{b}](A, \boldsymbol{v})$ and $P[c, \boldsymbol{d}](C, \boldsymbol{d})$, $A \preceq C$.

The definition of $\circ$ can get a bit complicated, depending on whether we think co-extensive mixed quantities can be distinct. If we think not, then we can define it by:

$P[a, \boldsymbol{b}] \circ P[c, \boldsymbol{d}] = P[A \circ C, \boldsymbol{v}]$ for any $\boldsymbol{v}$ where $P[a, \boldsymbol{b}](A, \boldsymbol{v})$ and $P[c, \boldsymbol{d}](C, \boldsymbol{d})$.

These defined relations will obey the same axioms, on their respective domains, as the original relations over the basic quantities do.[17]

Second, by comparativist lights, particular mixed quantities are not fundamental. In fact, they are not even F-intrinsic. Whether or not c and d instantiate $P[a, b]$ depends on how c and d relate to a and b. But these quantities are not themselves fundamental, and so don't need to be preserved by fundamorphisms. As a result, the mere fact that $P[a, b](c, d)$ does *not* mean that, if f is a fundamorphism, $P[a, b](f(c), f(d))$.

[16]When the basic quantities aren't extensive quantities — as discussed in note 2 — the intrinsic structure can get more complicated.

[17]There are devils in the details, of course. The structure of the masses is somewhat unlike the structure of the charges, for instance, and the space of mass-charges ends up being more charge-like than it is mass-like. But these details can be saved for another time.

There is, however, an F-intrinsic relation in the neighborhood: the *four-placed relation* that a, b, c, and d stand in whenever c and d instantiate $P[a, b]$. Since "$P[\_, \_]$" was defined using only fundamental notions, if $P[a, b](c, d)$ and f is a fundamorphism, $P[f(a), f(b)](f(c), f(d))$. These are different claims, and need to be kept distinct. Put glibly: By the comparativist lights, the mixed quantities are not F-intrinsic, but the comparative relations between them are.

Third, as I have defined them, mixed quantities are *higher-order relations*. But they don't have to be. Any mixed quantity has a **demotion**. For instance, a product $P[\boldsymbol{a}]$ has a demotion $P^{\downarrow}[\boldsymbol{a}]$ defined by

$P^{\downarrow}[\boldsymbol{a}](\boldsymbol{x})$ iff there are properties $\boldsymbol{b}$ where each x_i instantiates b_i and $P[\boldsymbol{a}](\boldsymbol{b})$.

Demotions of quotients are defined similarly. The demotion of a mixed quantity is the relation that holds between some things exactly when its higher-order counterpart holds between the (relevant) quantities instantiated by those things. It's a choice of no consequence as to whether we identify mixed quantities with products and quotients or instead their demotions.

Finally, much of our discussion of mixed quantities has been in service of nomic constants. But they can do more work for us than that. Consider a view, for instance, according to which mass-densities are fundamental, and masses are not. We would still want to know, on such a view, how to think about masses. But we can easily think of them as mixed quantities — products of mass densities and volumes (or three lengths). On such a view, claims about mass are ultimately claims about ratios holding between mass-densities and volumes.

4.5 Back to the Rocket!

The theory of mixed quantities from the last section reduces them to ratios between individual quantitities. Let's call it **Rational Reductionism**. It does not answer the Pandora argument on its own. To do that, we must couple it with **Nomic Relationalism**, the Baker-Jacobs thesis that (fundamental) nomic constants are fundamental relations. But — as we saw with the Z-fields — Nomic Relationalism needs to be supplemented with laws that connect the fundamental constants to the right kind of mixed quantities. That's why Nomic Relationalism needs Rational Reductionism.

To see how this answers the Pandora argument, lets formulate the laws intrinsically. Since the escape velocity law is derived from Newton's more general gravitational law, we'd best work with the latter. It's typically written:

$$F_g = G\frac{m_1 m_2}{r^2}.$$

Even dealing with this law is a bit much, though. It is part of the larger system of Newtonian mechanics, in which component forces act on an object to produce a net force, which then determines motion. I don't want to get derailed by issues about adding and subtracting forces, so let's instead focus on a special case of the law:

Simplified Newtonian Gravity: If x is an object experiencing no other forces and is at a distance r from an object of mass M, then its acceleration in meters per second squared towards the other object is equal to $\frac{GM}{r^2}$.

This is a standard textbook way of writing the law, in which "r", "M", and so on are variables to be filled in with numbers. Let's rewrite this in representationalist friendly terms, so that these symbols instead stand for properties. Where *si* is the SI system of units and G_{SI} is the value of the gravitational constant in $m^3/kg \cdot s^2$, the rewriting gives us

Rewritten Newtonian Gravity: If x is an object experiencing no other forces and is at a distance r from an object of mass M, then for any d and t, it will accelerate at a rate of one d per t^2 iff $si(d)/si(t)^2 = G_{SI} si(M)/si(r)^2$.

If we take that final formula and rewrite it, solving for G_{SI}, it becomes:

$$G_{SI} = si(d)si(r)^2/si(M)si(t)^2.$$

According to Nomic Relationalism, the gravitational constant is a fundamental relation, *Grav*. Furthermore, one fact about *Grav* is that $G(d, r, M, t)$ exactly when $G_{SI} = si(d)si(r)^2/si(M)si(t)^2$. So we can rewrite the law intrinsically:

Inrtinsic Newtonian Gravity: If x is an object experiencing no other forces and is at a distance r from an object of mass M, then it will accelerate at a rate of one d per t^2 where $Grav(d, r, M, t)$.

This gets us the first, "dynamic" portion of the law. But *Grav* needs to be associated with a mixed quantity, too. To get that, we have a second law,

***Grav* Dimensionality:** There are distances d and r, mass M, and duration t such that, for any d', r', M', and t',

$$Grav(d', r', M', t') \text{ iff } P[d, r, r/M, t, t](d', r', r'/M', t', t').$$ [18]

We can then explain why the gravitational constant gets the numerical representation it gets in any system of units. It gets that number because it's tied to a mixed quantity which gets that number.

Now let's consider Earth and Pandora. On Earth, the *Luna 1*'s velocity v_e equals the needed escape velocity. That velocity is given in SI units by

$$si(v_e) = \sqrt{\frac{2G_{SI}si(M)}{si(r)}}.$$

[18] As with $V^{\times}$, this can also be written to avoid existential quantification; see note 12 above.

Solving for G_{SI} and rewriting v_e as d/t, we get that

$$\frac{si(r) \cdot si(d)^2}{(2si(M)) \cdot si(t)^2} = G_{SI}.$$

Note that $2si(M) = si(M) + si(M) = si(M \circ M.)$ Given what we've said above, this means that the *Luna 1* is moving at a rate of one d per t where $Grav(r, d, M \circ M, t)$.

Now take any candidate fundamorphism f; since the distances and times aren't changed in Pandora, the rocket in Pandora must be moving at a rate of one $f(d)/f(t)$. But if it were the case that $Grav(f(r), f(d), f(M \circ M), f(t))$ then — since Pandora obeys the same laws — the rocket would make it into orbit. Since we know the rocket is going to crash, it must not be the case that $Grav(f(r), f(d), f(M \circ M), f(t))$. So the worlds are not fundamental duplicates after all.

Note here that we're not simply *asserting* that there is a fundamental difference between the worlds. We're using what, by the comparativist's lights, are intrinsic formulations of the laws, plus the setup of the case, to *show* that there is no fundamorphism between Earth and Pandora.

5 Ratio Equivalence and Cross-Family Comparisons

I claimed above that the basic representationalist setup let us define a ratio equivalence

$$a{:}b \equiv c{:}d,$$

where a and b must be in the same family, and c and d must be in the same family, but the first pair and the second pair can be (but don't have to be) from different families. This claim is not at all obvious, though, and even though I tried to waylay suspicion, readers may fairly remain skeptical.[19] This section is meant to make good on this claim.

To understand the definition, we need to understand how representationalists prove their representation theorem. I won't try to prove it here — there are too many "i"s to dot and "t"s to cross — but I'll sketch the basic idea.

This matters, because the definition to be given essentially says, "a, b, c, and d meet a condition ϕ", where ϕ guarantees that the theorem will assign the same ratio to a and b as it does to c and d. Unless we understand how the proof works, we won't understand how ϕ guarantees what is supposed to guarantee. But the condition ϕ will be stateable in the fundamental representationalist ideology (plus a bell and a whistle) without any special fundamental cross-family structure.

[19] Perry (2024: 12) raises related concerns, suggesting that we cannot even define "x is n-times as massive as y" without first proving the representation theorem. This section addresses the most general worry; see note 23 on Perry's more specific complaint.

5.1 *The Idea Behind the Proof*

My sketch follows Hölder's original proof in its essentialls.[20] We'll use the example of mass to illustrate the idea. We can think of our fundamental higher-order relations — $\preceq$ and $\circ$ — as corresponding to empirical operations: $a \preceq b$ holds iff, when we place an *a* and *b* thing on opposite sides of a balancing scale, the *a* side does *not* go down. (They may balance, so the *b* side may go down.) And $a \circ b = c$ iff, when we put a *c*-thing on one side of the scale, and both an *a*- and a *b*-thing on the other, the scale balances.

With this in mind, suppose we're handed two weights, *A* and *B*, and asked to find the ratio between them. How would we proceed?

Start by putting *A* on one side of the scale and copies of *B* on the other, one at a time. At some point the scale will tip to no longer favor the *A*-side. Call the mass *just before* this happens the **last full *B*-stop before *A***. Write down how many copies of *B* it took to get to this last full stop; we'll abbreviate this number as [1]. (If *B* started heavier than *A*, then $[1] = 0$.) One thing we now know is that the *A*-*B* ratio is greater than [1] but no greater than $[1] + 1$.

Next, cut the copies of *B* in half, and repeat the process. When we find the number of half-*B*s just before the scale tips, we call that mass the **last half-*B* stop before *A***, and write the number of half-*B*s it took to get there [2]. We can now also conclude that the *A*-*B* ratio is less than $\frac{[2]}{2}$ but no less than $\frac{[2]+1}{2}$.

Now we divide *B*-weights into thirds, and repeat the process, getting the **last third-*B* stop before *A*** and writing the number of third-*B*s in it as [3]. We then repeat this for every natural number *n*. When we are done, we know that the ratio is less than $\frac{[n]}{n}$ and not less than $\frac{[n]+1}{n}$, for every *n*.

Now take all the rationals of the form $\frac{[n]}{n}$. There will be no largest one of these; since these rationals are always lower than the actual ratio, there's always room to squeeze in another rational before we get to the ratio. But these rationals will have a least upper bound. That will be the first real number *greater* than all the rationals which are *smaller* than the ratio. So that least upper bound will, in fact, be the ratio.[21] This method thus delivers, for every pair of masses *A* and *B*, a ratio — which gives us the representation theorem's ratio function discussed in section 1.

[20] It is also very similar to Dewar's (2021: §§2) sketch, although with a different example quantity.

[21] The much-cited Krantz 1971 uses weaker axioms where we can't assume that any *B* can be divided into *n* equal parts. In those proofs we instead squeeze the actual ratio between "upper" and "lower" approximations of it, where the approximations are constructed simlarly to how we found each $[n]/n$. Despite the added complexity, a variation of the definition of $\equiv$ to be given ought to work in that setting, too, by establishing equivalent sequences of approximations. See e.g. Sider 2020: 131 and Field 1980: 122.

5.2 *Defining Ratio Equivalence*

The ratio between A and B will be the same as that between C and D exactly when the rationals that determine the A-to-B ratio are the same rationals that determine the C-to-D ratio. And *that* will happen exaclty when, whenever we divide B and D into an equal number of parts, the last B-part-stop before A is made up of just as many B-parts as the last D-part-stop before C. This is essentially what we need to define for our condition ϕ.

Before we start we need to settle on some logical resources. The axioms needed to prove the representation theorem aren't first-orderizable (Suppes et al., 2006: 227–229), so we'll need resources stronger than the first-order ones. One fairly weak resource that will do the job is an **ancestral operator**.[22] We attach this to any two-place open formula to get that formula's transitive closure. For instance, when we attach the ancestral operator to "x is a parent of y" we get the *ancestor* relation. For any formula ϕ, we write its ancestral as ϕ^*.

For now, we'll also use a "no more than" quantifier, letting us say ⌜there are no more ϕs than ψs⌝. We can use this to define "just as many": there are just as many ϕs as ψs if and only if there are no more ϕs than ψs and also no more ψs than ϕs.

Our sketch above talked about copies of a fraction of b. This is understood in terms of the ancestral. For any mass e, the **multiples** of e include e itself, $e \circ e$, $e \circ (e \circ e)$, and so on. The multiples of e are the masses in the ancestral of "concatenated with e". More precisely, f is a multiple of e iff either $f = e$ or e bears $(x \circ e = y)^*$ to f. We call e a **fraction of *b*** iff b is a multiple of e.[23]

If e is a fraction of b, we need to talk about the "last e-stop before a". This is done using concatenation and the ordering of quantities:

> ***c* is the last *e*-stop before *a*** iff c is a multiple of e and
>
> $$c \prec a \preceq c \circ e.$$

Thanks to how the proof works, the representation theorem will assign the same number to the a-b ratio as it assigns to the c-d one exactly when, if we divide b and d into the same number of parts, there will be just as many fraction-of-b stops before a as there are fraction-of-d stops before b. We use this fact to define our ratio equivalence.

Say that b and e **co-divide** with d and f iff b is a multiple of e, d is a multiple of f, and there are just as many multiples of e no greater than b as there are multiples of f no greater than d. Intutively, this means that e and f are the results of dividing b and d, respectively, into the "same number" of pieces. Now we define:

[22] If we want to ensure that there are as many quantities of a given type as there are real numbers, we need something stronger; in that case, plural quantification will do, and we can define the ancestral in terms of plural quantification.

[23] This is enough to let us address Perry's concern from note 19: "x is n times as massive as y" is just "x is a multiple of y, and there are only n multiples of x no greater than y."

a:b $\equiv$ c:d iff, for every e and f where b and e co-divide with d and f: if a^- is the last d-stop before a and c^- is the last f-stop before c, then a^- and e co-divide with c^- and f.

Note here that we never needed to assume that a and b were from the same family as c and d. Nor did we ever need to order or concatenate anything on the a-b side of the definition with anything on the c-d side. The only cross-family comparisons are done by the "just as many" quantifier.

5.3 *Worries about "Just As Many"*

Some may complain about the "just as many" quantifier on the grounds that it somehow smuggles in numbers, violating the representationalist vision of a number-free treatment of quantities. This concern strikes me as misguided; as Enderton (1977: 128–129) points out, a preschooler can determine whether there are just as many houses as people on a worksheet, even if there are more houses and people than they know how to count, by drawing a line connecting each person to a unique house. Still, with a bit of ingenuity we can avoid even the appearance of impropriety by getting rid of "just as many", too.

We only used "just as many" in the definition of "co-divide"; to get rid of the former, we need only redefine the latter. If we're allowed ordered pairs of quantities, it is easy to do. Given pairs $\langle a, b\rangle$ and $\langle a', b'\rangle$, we define $\langle a, b\rangle \circ \langle a', b'\rangle$ as $\langle a \circ a', b \circ b'\rangle$. This "pair-concatenation" will then have an ancestral. To say that a and e co-divide with c and f is just to say that $\langle a, c\rangle$ bear the ancestral of "$x \circ \langle c, f\rangle = y$" to $\langle c, f\rangle$.

Of course, if "just as many" is too mathematical to take for granted, orderd pairs will be, too. But we can use a simple variant of the trick pioneered by Burgess et al. (1990) to trade the pairs for fusions of quantities. Assume all quantities are atomic, and let a dyad be a fusion of no more than two quantities. The **flavor** of a dyad is given by the families of its parts. Two dyads are **cohesive** iff they have the same flavor. Furthermore, a dyad is **monogustic** iff both of its parts are from the same family.

Now we can define concatenation of cohesive dyads. Notice that, when it comes to monogustic dyads, of their two parts, one will be no greater than the other. The concatenation of two monogustic dyads is the fusion of the concatenation of their smaller parts with the concatenation of their larger parts. That is, if X is the fusion of a and b, Y is the fusion of c and d, $a \preceq b$, and $c \preceq d$, then $X \circ Y$ is the fusion of $a \circ c$ and $b \circ d$. When two dyads are instead digustic, their concatenation is the fusion of their same-family parts.

This gives us another defined concatenation relation, which has an ancestral. In this case, a and e co-divide with c and f exactly when there is a dyad D fusing a and c and a dyad F fusing e and f where F bears the ancestral of "concatenate with F" to D.

These definitions use neither numbers nor sets. And clearly the only cross-quantity structure is mereological, used to construct dyads. No spooky, novel connections between quantity families are needed.[24]

6 Mixed Quantities are Everybody's Responsibility

Thus far I've focused primarily on comparativism and its woes. But one needn't be a comparativist to worry about mixed units and the role of constants. Representationalism aims to understand, in a number-free way, what statements using unit-relative numbers are saying about the world. Some of these statements include formulations of the laws of nature, which use numbers to talk about nomic constants. Having put in the work to explain "7 kg", it seems a pity to stop before giving "$6.67 \times 10^{-11}\ m^3/kg \cdot s^2$" the same treatment. (Jacobs, 2023: 799–800)

Happily, the absolutist can crib the comparativist's homework. Rational Reductionism works just as well for absolutists as it does for comparativists.

That doesn't mean everything carries over in exactly the same way. According to comparativists, products and quotients are not F-intrinsic. According to absolutists, they are. If a and b are fundamental properties, since $\equiv$ is defined in terms of the fundamental and $P[a,b]$ defined in terms of $\equiv$, $P[a,b]$ will be F-intrinsic.

This suggests that absolutists can more or less *identify* the gravitational constant with its associated quotient. Let d, r, M, and t be quantities that, according to the comparativist, stand in *Grav* together. Then the absolutist can *define Grav* by

$$Grav(d', r', M', t') \text{ iff } P[d, r, r/M, t, t](d', r', r'/M', t', t'),$$

Since the quantities are fundamental, the thus-defined *Grav* will be F-intrinsic. The absolutist can then go on to endorse (for instance) Intrinsic Newtonian Gravity, and her theory will give rise to exactly the number-using formulations of the laws that she wants it to.

(On this view, could the gravitational constant been different? Not *de re*, but *de dicto*: a different product could have played the gravitational-constant role.)

Not every absolutist will be happy with this move. Suppose $P[d, r, r/M, t, t](a, b, b/c, e, e)$. Then we could have instead defined *Grav* by

$$Grav(d', r', M', t') \text{ iff } P[a, b, b/c, e, e](d', r', r'/M', t', t').$$

[24] I've restricted my attention to extensive quantities, but not all quantities we want to be representationalists about are extensive. While this will introduce complications, they ought not undercuts the basic idea behind the definition of "$\equiv$". As Wolff (2020: §6.2.3) argues, any quantity will at least have a "Super-Ratio" structure, and this structure ought to be enough to define a ratio equivalence relation for each pair of quantitiy families. This, however, is not the place to work out those fine details.

To some minds, this will mean that the gravitational constant is unacceptably arbitrary; why (they will wonder) did we choose *these* quantities to pick out our product, rather than some others?

Plausibly, "$P[d, r, r/M, t, t]$" and "$P[a, b, b/c, e, e]$" are simpy different names for the same relation. So the worry isn't about arbitrariness in the world; it's not as though we're taking one relation (such as *mass in kilograms*) as fundamental while declaring a distinct-but-equally-good relation (such as *mass in grams*) as non-fundamental. It is, rather, a worry about arbitrariness in the way that we *specify* this relation.

I expect disagreement about whether this kind of arbitrariness is objectionable or not. Those unbothered by the objection can ignore it. The rest can instead take refuge in Nomic Relationalism, positing another (non-arbitrarily-specified) fundamental *Grav* relation.

Rational Reductionism (with or without Nomic Relationalism) also answers Sider's (§2.2) claim that absolutists are in the same Pandora-shaped pot with comparativists. Our intrinsic formulation of Newtonian gravitation tell us exactly what needs to be different between the worlds for their respective rocket to do different things, so we know just what the laws are "seeing" that lets them distinguish the cases.

7 Algebraic Realism

I've spent the paper developing Rational Reductionism about mixed quantities. But it has a rival. Dewar (2021) develops a view I will call **Algebraic Realism**.[25] We ought to see how they stack up against each other. I'll describe the view in §7.1 and evaluate it in §7.2.

7.1 *The View*

The easiest way to introduce the view is by contrasting it with Rational Reductionism, and the contrast is easiest to see when both views are varieties of absolutism. So let's start there.

Absolutist Rational Reductionism starts with some basic fundamental quantities and defines mixed quantities as relations between them. These relations are F-intrinsic, but not fundamental.

Absolutist Algebraic Realism also starts with some basic fundamental quantities. But the mixed quantities are equally fundamental. Just as the basic quantities are connected, within families, by $\preceq$ and $\circ$ relations, the mixed quantities are connected

[25]On my reading, Dewar's (2021) paper admits two interpretations. One interpretation is the view to be described. On the other interpretation, Dewar characterizes Rational Reductionism in an "external" way, with an end-run around explicitly defining products and quotients. On the latter interpretation, this paper furthers Dewar's project by providing these explicit definitions, thus shoring it up against criticisms of the sort that Jacobs (2022) raises for such "external" approaches.

to each other by these relations. But they are also connected to the basic ones by other, cross-family relations. For example, there is a three-placed relation $*$ where, for any mass m and length l, there is a mass-length a such that $*(m, l, a)$. Since $*$ is functional, there is ony one such a, and we can write it as $m * l$. Just as $\circ$ and $\preceq$ obey certain princples, so does $*$. The central principle is:

$$m_1 * l_1 = m_2 * l_2 \text{ iff } m_1{:}m_2 \equiv l_2{:}l_1.$$ [26]

Absolutist Algebraic Realism is, of course, absolutist; whether it is *mixed* absolutist depends on whether $\preceq$, $\circ$, and $*$ are fundamental or instead somehow derived from the natures of the quantities themselves. There is also a comparativist version of the view, according to which $\preceq$, $\circ$, and $*$ are fundamental, but the determinate quantities (whether basic or mixed) are not.

Despite a different underlying metaphysics, Algebraic Realism and Rational Reductionism share many structural similarities. Algebrism reifies Reductionism's products as new, *sui generis* quantities, related to the basic ones (and each other) by $*$. For each defined Reductionist product P, there is a corresponding Algebraic sui generis quantity p, and an $*$-using formula where $\phi(a, b, p)$ if and only if $P(a, b)$. Given such a P and p, let's call P the **Shadow** of p, and p the **Form** of P. So, rephrasing: Algebraism reifies Reductionist Shadows as new, sui generis Forms.[27]

Given these structural similarities, it should come as no surprise that comparativist Algebraism must also answer the Pandora argument. Take the function that caused problems for our original comparativist. On Comparativist Algebraic Realism, it won't need to preserve (basic *or* mixed) quantities to be a fundamorphism. It *will* need to preserve structural relations between quantities, including the new $*$ relation. But if a is the Form of the actual gravitational constant, f will map it to a "permuted" quantity that bears the same higher-order-relations ($*$ included) to f's values for other quantities.

Nomic Relationalism, converted to the key of Algebraic Realism, becomes the thesis that there is a fundamental higher-order property *Grav* that applies to exactly one mixed quantity — the Form of the quotient that Reductionists associate with *Grav*. If this Form gets mapped to another under a structural-relation-preserving mapping, and if *Grav* plays the same role in the laws of nature in both worlds, then f won't perserve *Grav* and so won't be a fundamorphism.

[26]See Dewar (2021) for a full development of this view; the version I present corresponds, on this interpretation of Dewar, to his second option, in which we start not with "pure ratio" quantities but instead what I've called the "basic" quantities. The largest difference between my brief sketch and Dewar's presentation is that he makes no attempt to characterize the algebraic relations without appealing to numbers. But I strongly suspect that, using the tools developed here, this can be done.

[27]Despite the evocative terms, I mean to imply no deep similarities betewen Algebraism and any Platonic doctrine.

What *is* this role in the law of nature? If Q is the quotient corresponding to the gravitational constant and q its Form, there will be a formula ψ where

$$\psi(d, r, M, t, q) \text{ iff } Q(d, r, M, t).$$

The Algebraist can then formulate a law:

Intrinsic Algebraic Gravity: If x is an object experiencing no other forces and is at a distance r from an object of mass M, then it will accelerate at a rate of one d per t^2 where, for some a, $\psi(d, r, M, t, a)$ and $Grav(a)$.

A second law will then tell us that *Grav* is instantiated by exactly one mixed quantity a of the right type.

As before, the absolutist is under no direct challenge from Pandora. They may simply identify the gravitational constant with a particular algebraic quantitiy q. Since they take q to be fundamental, f will have to preserve it to be a fundamorphism. They may still, however, worry about Sider's indirect challenge. If so, they can answer it with a variant of Intrinsic Algebraic Gravity above, one which removes quantification over a and replaces the final clause with "$\psi(d, r, M, t, q)$ and $Grav(q)$".

7.2 *Comparative Evaluations*

This is the section where I'm supposed to trimphantly extol the virtues of Reductionism over the vices of Algebraism. Given the strong structural similarities between the views, though, I can't do that in good conscience. I do think the scales tip in favor of Rational Reductionism, but it's a thin case, and reasonable people can disagree.

Let's start with Algebraism's prospective advantages.

7.2.1 Algebraism's Prospective Advantages

Prosepctive Advantage One: Absolutist Algebraism doesn't face the specification worry discussed in §6. Even if we can't talk about a particular Shadow $P[a, b]$ without going through some specific a and b, the Form of this Shadow is a *sui generis* entity in its own right, needing no definition. Whether we think this is much of an advantage will depend on how pressing we found the original worry.

Prosepctive Advantage Two: Consider products $P[m, l]$ and $P[l, m]$ of a length l and a mass m. On our usual understanding of relations, these are *different*; after all, in one of them, the mass has to come first, and in the other, the length has to come first. On the other hand, the algebraist can identify both of these as shadows of a *single* form, one that relates to length-mass pairs but not in any particular "order".

This prospective advantage might be spun in any of several ways. On one version, it's just *weird* that there should be two such relations. On another, it's too ontologically profligate: We're doing work with two relations that we should do with just one.

The advantage, spun in either of these ways, is illusory. The Algebraist should admit that the relations $P[m,l]$ and $P[l,m]$ are both *there* — we can define them from ideology the Algebraist accepts, after all. The Algebraist neither gains economy nor excises weirdness by adding new, sui generis quantities.

A third way of spinning the advantage is stronger, though. Consider the Reductionist's *Grav* Dimensionality law. It uses a particular Shadow, $P[d,r,r/M,t,t]$. But why should the law be formulated using *this* relation, rather than an equivalent one with the arguments permuted? Certain mindsets will find formulations of laws that appeal to arbitrary choices of entities — such as this relation, rather than some equivalent one — objectionable, and they may prefer Algebraism as a result.

This objection to Reductionism lies close to the specification worry of §6, but it isn't precisely the same. If $P[a,b](c,d)$, it's very tempting to think that $P[a,b] = P[c,d]$. So the worry of that section is about which words we use to specify a *single relation*. Plausibly, however, the relations $P[m,l]$ and $P[l,m]$ are *distinct relations*, as are the Shadow P used in *Grav* Dimensionality and some P^* that permutes its argument places. One could, in principle, not mind arbitrariness in how our formulations of the laws pick out various relations while still recoiling at arbitrariness in which relations are thereby picked out.

The Reductionist does have precedent to argue that, despite appearances, P and P^* (and other permuted Shadows) are identical after all (Williamson, 1985; Fine, 2000). This isn't the place to dive deeply into the metaphysics of relations, so I'll leave the matter here.

7.2.2 Algebraism's Prospective Disadvantages

Prospective Disadvantage One: Strong comparativists will have no use for Algebraism.

I suggested in §1.1 that, if relationalists (which include strong comparativists) could solve their other problems, they could make use of Rational Reductionism. Then I dropped the issue, ignoring strong comparativism entirely from thence on. Note, though, that the strong comparativist's main problem is finding enough concrete objects to go proxy for properties or points-of-quantity-space in the representation theorem. If they can do that, they presumably can define counterparts of ratio equivalence. Ratio equivalence then lets them say (for instance) when two massive objects "count" as the product of two other massive objects. Having done that, they can answer the Pandora argument by positing a fundamental comparative relation *R-Grav* that holds between concrete objects exactly when, as we substantivalists would say, *Grav* holds between the quantities that those objects bear.

So reductionism has at least the potential to help relationalists, including strong comparativists. Algebraism not so much, for the simple reason that it does its thing by positing exactly what the relationalist wants to eliminate: *sui generis* quantities. That is, of course, a parochial concern that will cut little ice with substantivalists. Still, relationalists in the market for a theory of mixed quantities will have no use for

Algebraism.

Second Prospective Disadvantage: Algebraism can seem oddly roundabout. Compare its formulation of Intrinsic Newtonian Gravity with the Reductionist's. Both explicitly reference the quantities d, r, M, and t. The Reductionist stops there, applying the *Grav* relation to these quantities. The algebraist has to go one step further, relating these quantities to the further, mixed quantity q that instantiates *Grav*. But q only gets into the picture by its relation to the other, basic quantities. It's the basic quantities that hook directly onto chunks of the world, not the mixed ones.

An Algebraist may respond by identifying q with a relation between the things that have d, r, M, and t. In doing this, she makes her mixed quantities march in step with the Reductionist's demotions (§4.4). Spheres vanish in a Z-field, for instance, because they bear a certain mass-squared relation $v^{\times}$ to each other.

This takes the roundaboutness out of the laws, but not the theory. *What it is* for two spheres to bear the mass-squared relation to each other is just for their respective masses to bear a certain relation to each other. This fact about square masses is hidden when we treat a square mass as a sui generis entity bearing $*$-relations to masses.

This worry is especially pressing for comparativist and mixed-comparativist variants of the view. According to these views, $*$ is itself a fundamental relation, which suggests it ought to be adding some new structure to the space of quantities. But since the Algebrist can define the demotions of quotients without using $*$, this new "extra" structure is masking what we already have. In this case, $*$ looks like more fundamental ideology than we in fact need.

Prospective Disadvantage Three: The third disadvantage is a direct corrolary to the second. Not only is the $*$-relation redundant, so are the Algebrist's posited Forms, too. They are *extra*, doing no work that could have been done just with the Shadows.

I have no uncontroversial formulation of Occam's Razor, but it's generally accepted that redundant entities should be jettisoned.[28] In the presence of our definable products and quotients, the Algebraist's basic mixed quantities — its Forms — are redundant. Everything they do, the Shadows do, too. Unless there is some additional theoretical benefit to Algebraism that Reductionism can't mirror, we ought to do away with it.

8 Conclusion

Representationalism gives us an attractive story about what numbers have to do with quantities, but that story isn't complete until it tells us what numbers have to do

[28]This isn't the same as saying that if T posits fewer entities than T', T is to be preferred to T'; redundancy is a stronger condition, which requires, *inter alia*, that each of T' entities is also in T.

with mixed quantities, such as the gravitational constant. Rational Reductionism elegantly extends the story, using the definability of ratio equivalence in the general representationalist framework to explain what these numbers represent.

By itself, that doesn't address Baker's Pandora argument. The argument targets the comparativist's claim that quantities aren't fundamental; being non-fundamental, quantities needn't be preserved by fundamental duplication. The solution is to add some fundamental metaphysical gizmo that locks onto the value of the nomic constant so as to keep Pandora from being a fundamental duplicate of Earth.

But we can't lock the gizmo to the value of the nomic constant until we know just what such a value consists in. The value can't be the *numerical* value, since (given representationalism) such a value is a mere representation for some underlying metaphysical structure. That's where Rational Reductionism comes in; it tells us just *what* that underlying metaphysical structure consists in. With that in hand, we can use Nomic Relationalism to tie it down fundamentally, blocking the argument.

This provides something of an argument for Rational Reductivism; comparativists can use it to answer an important challenge. But Reductive Relationalism is attractive in its own right. It extends the standard representationalist picture elegantly, adding no new metaphysics but simply exploiting what has been there all along. Representationalists had the resources to extend their story to mixed quantities from the get-go. It just took some work to spell that story out.

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